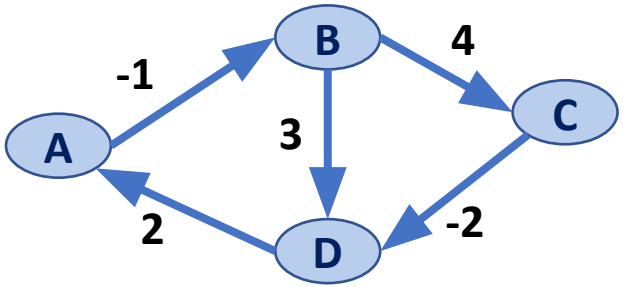
All Source Shortest Path

Learning Objectives

1. Formulate Floyd-Warshall as a Dynamic Programming Problem
2. Understand the Example Walkthrough

Problem Initialization

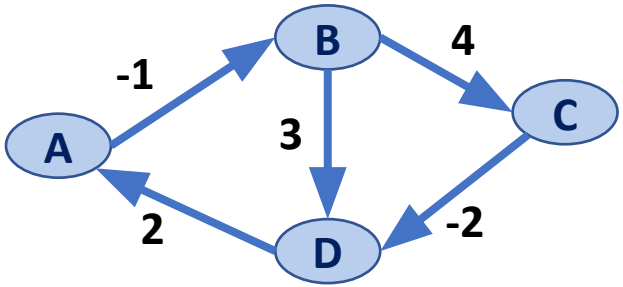
d	A	B	C	D
A				
B				
C				
D				



d is a distance matrix to track shortest paths
d[start][end] is the current shortest distance
from vertex start to vertex end

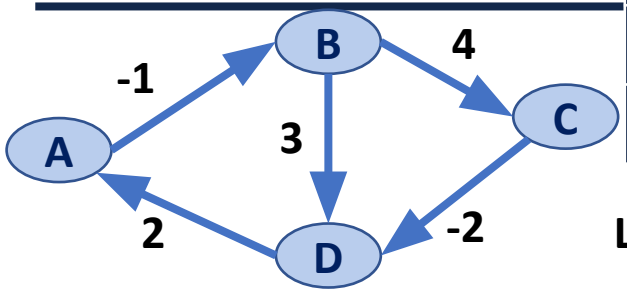
Problem Formulation

d^0	A	B	C	D
A	0	-1	inf	inf
B	inf	0	4	3
C	inf	inf	0	-2
D	2	inf	inf	0

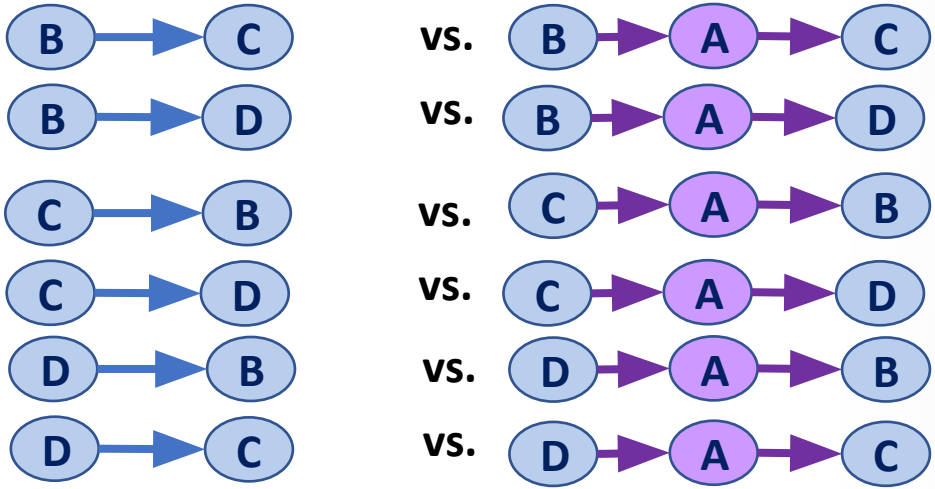


$d^{(k)}[start][end] = \min(d^{(k-1)}[start][end], d^{(k-1)}[start][via] + d^{(k-1)}[via][end])$

d^0	A	B	C	D	d^A	A	B	C	D
A	0	-1	inf	inf	A				
B	inf	0	4	3	B				
C	inf	inf	0	-2	C				
D	2	inf	inf	0	D				

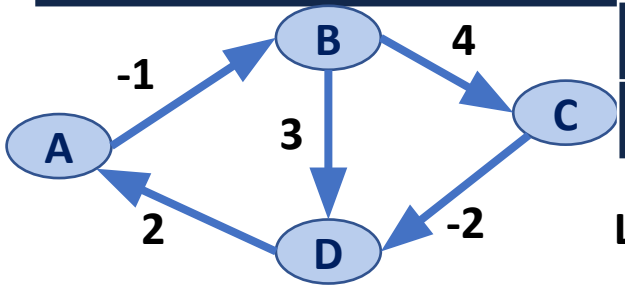


Let us consider via=A

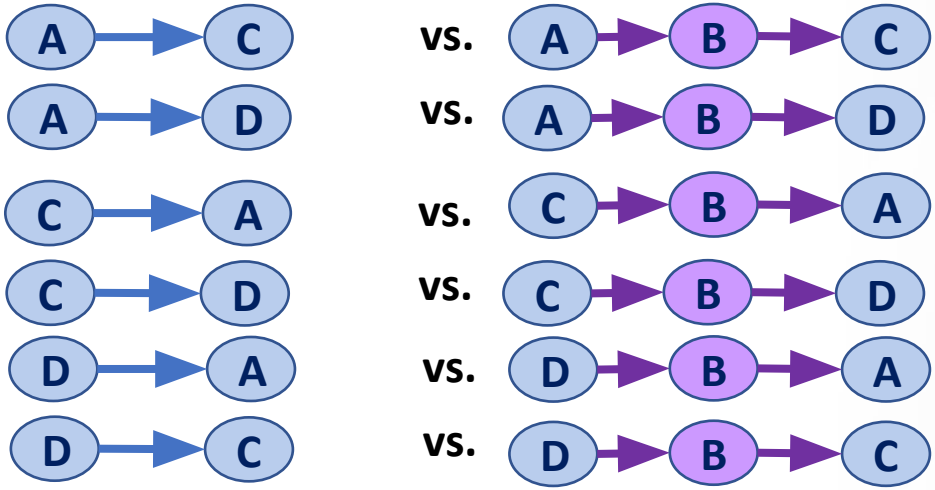


d^A	A	B	C	D
A	0	-1	inf	inf
B	inf	0	4	3
C	inf	inf	0	-2
D	2	1	inf	0

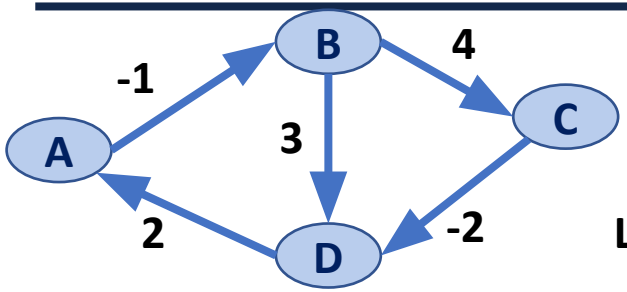
d^B	A	B	C	D
A				
B				
C				
D				



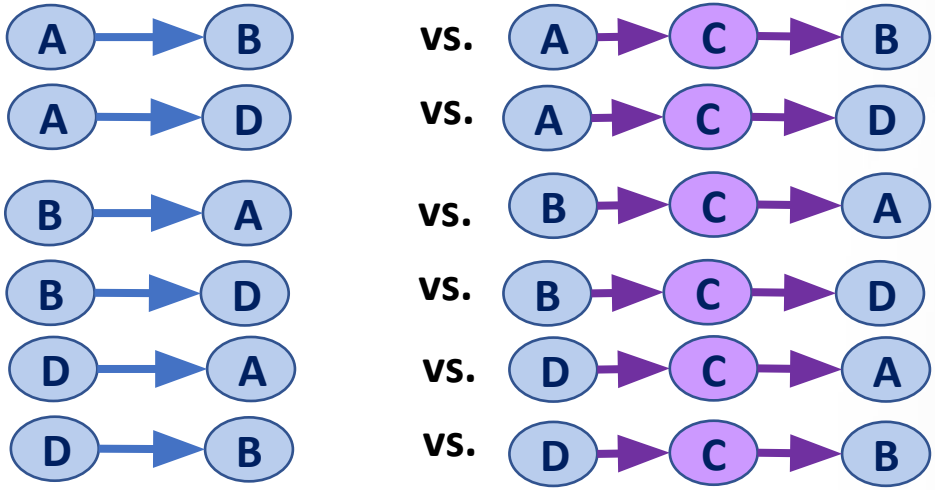
Let us consider via=B



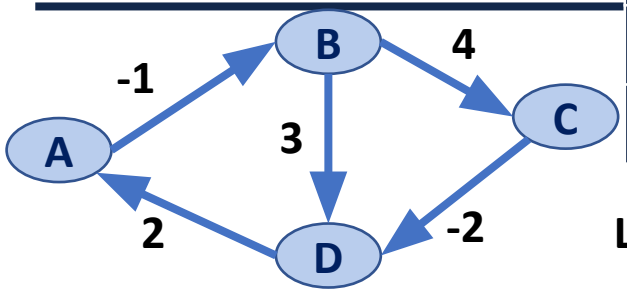
d^B	A	B	C	D	d^C	A	B	C	D
A	0	-1	3	2	A				
B	inf	0	4	3	B				
C	inf	inf	0	-2	C				
D	2	1	5	0	D				



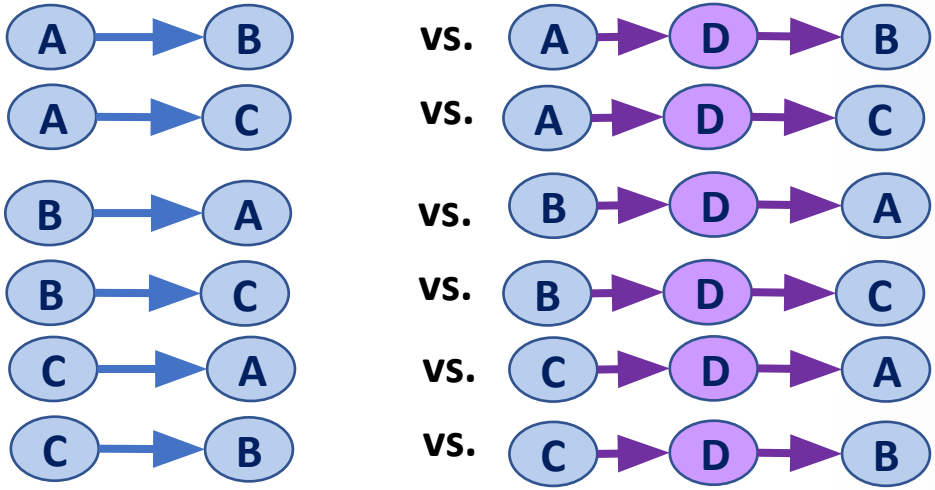
Let us consider via=C

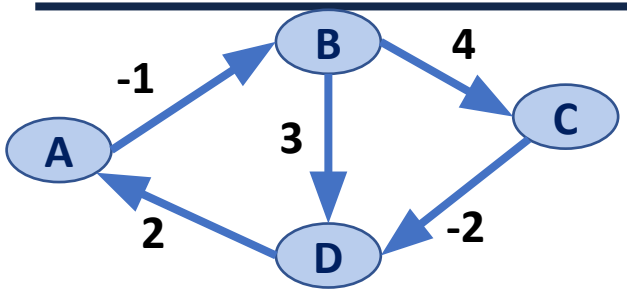


d^C	A	B	C	D	d^D	A	B	C	D
A	0	-1	3	1	A	0	-1	3	1
B	inf	0	4	2	B	5	0	4	2
C	inf	inf	0	-2	C	0	-1	0	-2
D	2	1	5	0	D	2	1	5	0



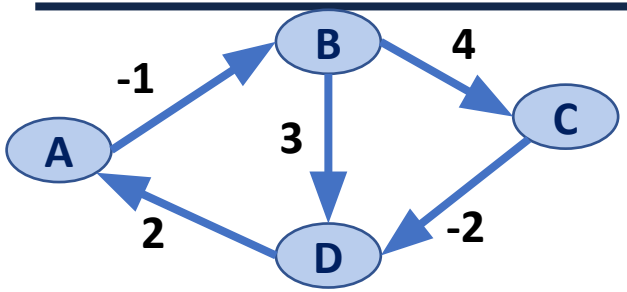
Let us consider via=D





d^D	A	B	C	D
A	0	-1	3	1
B	5	0	4	2
C	0	-1	0	-2
D	2	1	5	0

How do you track the path taken?



s	A	B	C	D
A	A	B		
B		B	C	D
C			C	D
D	A			D

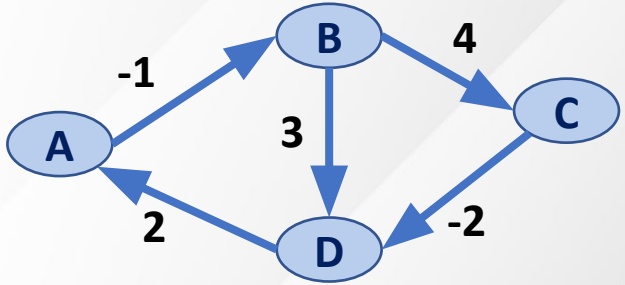
How do you track the path taken?
s is a matrix keeping track of the next vertex in the path. s[start][end] gives the next vertex in the path from vertex start to vertex end

s	A	B	C	D
A	A	B		
B		B	C	D
C			C	D
D	A			D

d^0	A	B	C	D
A	0	-1	inf	inf
B	inf	0	4	3
C	inf	inf	0	-2
D	2	inf	inf	0

d^A	A	B	C	D
A	0	-1	inf	inf
B	inf	0	4	3
C	inf	inf	0	-2
D	2	1	inf	0

Let us consider via=A

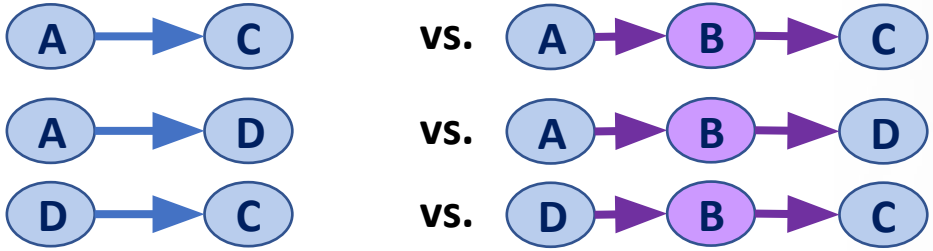


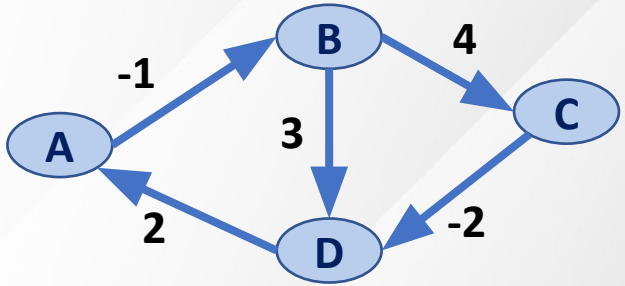
s	A	B	C	D
A	A	B		
B		B	C	D
C			C	D
D	A	A		D

d^A	A	B	C	D
A	0	-1	inf	inf
B	inf	0	4	3
C	inf	inf	0	-2
D	2	1	inf	0

d^B	A	B	C	D
A	0	-1	3	2
B	inf	0	4	3
C	inf	inf	0	-2
D	2	1	5	0

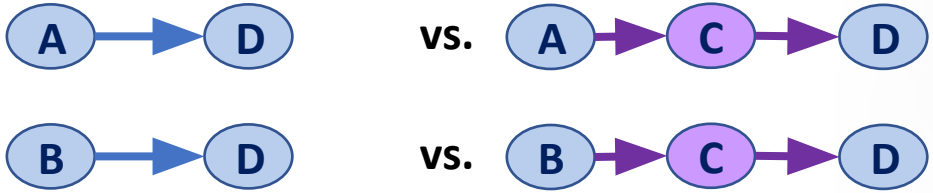
Let us consider via=B

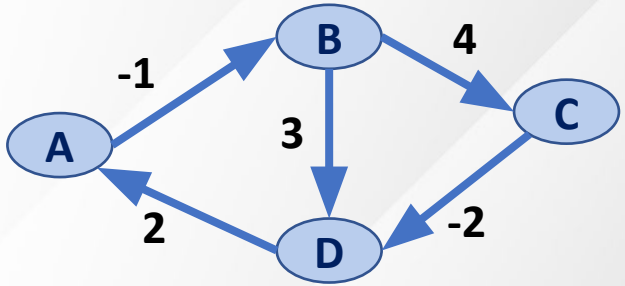




s	A	B	C	D	d^B	A	B	C	D	d^C	A	B	C	D
A	A	B	B	B	A	0	-1	3	2	A	0	-1	3	1
B		B	C	D	B	inf	0	4	3	B	inf	0	4	2
C			C	D	C	inf	inf	0	-2	C	inf	inf	0	-2
D	A	A	A	D	D	2	1	5	0	D	2	1	5	0

Let us consider via=C





s	A	B	C	D	d^C	A	B	C	D	d^D	A	B	C	D
A	A	B	B	B	A	0	-1	3	1	A	0	-1	3	1
B	D	B	C	C	B	inf	0	4	2	B	5	0	4	2
C	D	D	C	D	C	inf	inf	0	-2	C	0	-1	0	-2
D	A	A	A	D	D	2	1	5	0	D	2	1	5	0

Let us consider via=D